MA 441, Fall 2014

Assignment 1.

This homework is due *Thursday*, September 4.

There are total 33 points in this assignment. 30 points is considered 100%. If you go over 30 points, you will get over 100% for this homework and it will count towards your course grade (but not over 115%).

Collaboration is welcome. If you do collaborate, make sure to write/type your own paper and *credit your collaborators*. Your solutions should contain full proofs. Bare answers will not earn you much.

This assignment covers section 1.1 in Bartle–Sherbert.

(1) (Exercise 1.1.5 in textbook) Prove the Distributive laws:
   (a) [3pt] $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$,
   (b) [3pt] $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.

(2) (1.1.10) Let $f(x) = 1/x^2$, $x \neq 0$, $x \in \mathbb{R}$.
   (a) [1pt] Determine the direct image $f(E)$ where $E = \{x \in \mathbb{R} \mid 1 \leq x \leq 2\}$.
   (b) [1pt] Determine the inverse image $f^{-1}(G)$ where $G = \{x \in \mathbb{R} \mid 1 \leq x \leq 4\}$.

(3) Let $f : A \to B$ and $E, F \subseteq A$.
   (a) [3pt] (Part of 1.1.14) Show that $f(E \cup F) = f(E) \cup f(F)$.
   (b) [3pt] (Part of 1.1.14) Show that $f(E \cap F) \subseteq f(E) \cap f(F)$.
   (c) [2pt] Show that not always $f(E \cap F) = f(E) \cap f(F)$. (*Hint:* to find a counter-example, you can start by picking E and F that do not intersect *at all*.)
   (d) [2pt] Show that not always $f(E \setminus F) \subseteq f(E) \setminus f(F)$. (*Hint:* to find a counter-example, you can start by picking $f(E)$ and $f(F)$ that *coincide*.)

(4) (Part of 1.1.15) [3pt] Let $f : A \to B$ and $G, H \subseteq B$. Prove that $f^{-1}(G \cap H) = f^{-1}(G) \cap f^{-1}(H)$.
   Comment. Compare to 3c.

(5) (1.1.22+) Let $f : A \to B$ and $g : B \to C$.
   (a) [3pt] Show that if $g \circ f$ is injective, then f is injective. Give an example that shows that g need not be injective.
   (b) [3pt] Show that if $g \circ f$ is surjective, then g is surjective. Give an example that shows that f need not be surjective.

(6) (1.1.19)
   (a) [3pt] Show that if $f : A \to B$ is injective and $E \subseteq A$, then $f^{-1}(f(E)) = E$. Give an example that equality need not hold if f is not injective.
   (b) [3pt] Show that if $f : A \to B$ is surjective and $H \subseteq B$, then $f(f^{-1}(H)) = H$. Give an example to show that equality need not hold if f is not surjective.